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| |  |  | | --- | --- | |  | Name:  **Year 12 Mathematics Methods**  **Test 6:** Random Sampling, Sample Proportions and Confidence Intervals for Sample  Proportions  **Reading Time:** 2 minutes  **Working Time:** 40 minutes  **Materials:** CAS calculator, Formula Sheet and 1 page of notes **38** **Marks** | |

Question 1 (7 marks)

From a random sample of people, it was found that 54 of them subscribe to a streaming music service. A symmetric confidence interval for the true population proportion who subscribe is .

(a) Determine the value of , by first finding the mid-point of the interval. (3 marks)

(b) Determine the confidence level of the interval. (4 marks)

Question 2 (9 marks)

The management at a conference centre was concerned about the quality of the free pens that it provided in its meeting rooms. A staff member tested a random sample of 150 pens and found that 18 of them fail to write.

(a) If is the true proportion of pens that fail to write and is the corresponding sample proportion, use the above sample to determine

(i) . (1 mark)

(ii) the approximate margin of error for a 98% confidence interval for . (3 marks)

(iii) an approximate 98% confidence interval for . (1 mark)

(b) The stationery company that supplies pens to the conference centre claim that no more than 3 in 50 pens fail to write. Use your previous working to comment on the validity of this claim. (2 marks)

(c) Comment on how the margin of error would change in (a) (ii) if

(i) the quality of the pens had been better. (1 mark)

(ii) the required level of confidence decreased. (1 mark)

Question 3 (7 marks)

A student planned to investigate what proportion of the 1260 students at their school had access to more than one computer at home.

(a) The student thought of the following three ways to select a sample from the population. Briefly discuss the main source of bias in each method.

(i) Wait at the bus-bay after school and ask the first 50 students who show up.

(1 mark)

(ii) Advertise the survey in a whole school assembly and ask the first 50 students who volunteer to stay behind. (1 mark)

(iii) Select and ask every 100th student from the school roll. (1 mark)

(b) Assuming that 80% of students had access to more than one computer at home, the student carried out 100 simulations in which a sample proportion was calculated from a random sample of 64 students.

(i) Explain why it is reasonable to expect that the distribution of the sample proportions would approximate normality. (2 marks)

(ii) Determine the mean and standard deviation of the normal distribution that the sample proportions would approximate. (2 marks)

Question 4 (7 marks)

(a) (i) Sketch a histogram of a binomial distribution with . (1 mark)

(ii) Sketch a graph of the distribution of the sample means when 20 samples

of size 30 are drawn from a population that has a mean of 10. (2 marks)

(b) Two students took random samples from the Year 11 girls at the local high school and noted the number of girls that had blonde hair.

Student 1 sampled 10 students and found 2 blondes.

Student 2 sampled 100 students and found 23 blondes.

(i) Which student would be expected to have the best indication of the proportion of blondes in Year 11? Explain. (2 marks)

A third student, Student 3, took 10 different samples of 10 students and counted the number of blondes in each sample. He then averaged the averages of his samples and got 2.56.

(ii) Which of the three students would have had the best prediction result?

Explain. (2 marks)

Question 5 (8 marks)

A student repeatedly took random samples of size from a large population in which it was known that of people were left-handed. For each sample, the proportion of left-handed people was calculated and recorded as the sample proportion.

(a) Use an appropriate binomial distribution to determine the probability that the sample proportion is no more than in a randomly chosen sample. (3 marks)

(b) After recording a large number of sample proportions, the student used them to create a histogram from which the approximate normality of their distribution was evident.

(i) Determine the expected mean and standard deviation of the observed normal distribution. (2 marks)

(ii) Use this normal distribution to determine the probability that the sample proportion is no more than in a randomly chosen sample. (1 mark)

(iii) Describe how the parameters calculated in (i) would change if the student took larger random samples. (2 marks)